Isaac Newton's theory of inertially caused pressure resistance, reinstated
Flow separations and instability drag as the mechanisms of Newton's theory.
The theory's defeat by d'Alembert's paradox, each correct within conditions only differentiated in the 19th century. An amusingly convoluted history.

Preprint at: file:///Users/philiprandolph/Downloads/rcsdindex.html

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> Projectiles, of course, arouse motion in fluids by going through them, and this motion arises from the excess of the pressure of the fluid on the front of the projectile over the pressure on the back, and cannot be less in infinitely fluid mediums than in air, water, and quicksilver in proportion to the density of matter in each. And this excess of pressure…not only arouses motion in the fluid but also acts upon the projectile to retard its motion…[1] –Isaac Newton, *The Principia: Mathematical Principles of Natural Philosophy,* 1999 translation from the third edition of 1726.
>
> …the resistance…arises from the inertia of matter…[2] –Newton, *The Principia*.
>
> I do not see, I admit, how one can satisfactorily explain by theory the resistance of fluids…the theory, developed in all possible rigor, gives, at least in several cases, a strictly vanishing resistance; a singular paradox which I leave to future geometers for elucidation. –Jean le Rond d'Alembert, 1768.[34]

Abstract:

In his *Principia (*1687, 1713, 1726*),* Isaac Newton asserted his theory of inertially caused non-viscous (frictionless) pressure resistance (drag) within hypothetical inviscid (frictionless) fluids and as a component of drag within real fluids. Newton correctly asserted that drag, via pressures, makes a "motion" (Newton's term for momentum) exchange between object and fluid, slowing his "projectile" and accelerating its wake. But critically, although eddies behind rocks and bridge pylons were common knowledge, and despite his experiments with sinking objects, Newton didn't investigate flow disturbances and eddies behind moving objects in fluids. Had he, consistently with his "motion" exchange, Newton would have concluded that by Descartes' and Huygens' conservation of momentum and his three laws of "motion," addition of linear and angular momentum to flow disturbances would subtract momentum from the object via pressure forces, providing the mechanism and proof of his non-viscous inertial pressure drag. Wake disturbances were not formally treated until 1842 when Sir George Gabriel Stokes asserted that while 'steady' flows were one solution to the Navier-Stokes equations, eddies might develop. Numerous scientists then studied unstable flows, flow separations, wake vortices, turbulence, and resulting drag, even in inviscid fluids. These studies should have validated Newton's theory. Shortly after Newton's death, two preeminent mathematicians, Jean le Rond d'Alembert and Leonhard Euler, using independently derived versions of the Bernoulli equation, correctly proved that under conditions they assumed were real – inviscid fluids and the assumption that flows around a bluff fore-aft symmetrical object would be smoothly fore-aft symmetrical – drag would be zero. Inviscid and 'steady' conditions would eventually be shown to be extremely useful simplifying fictions for engineering computations, under which the real wake disturbances that are the mechanisms of Newton's theory can't exist. But in a time before wake disturbances were formally studied, it appeared that Newton's assertion of non-viscous pressure drag had been disproved. Newton's theory has remained dismissed, herein reinstated.

General prediction before specific mechanism: instability drag

It should be called instability drag. Or Newtonian non-viscous inertial instability pressure drag. But that's hindsight. Even though with his three laws of motion, experiments, and his profound physical intuition Newton predicted such drag, he couldn't know how it worked, nor even that he didn't know. Had he investigated the swirls behind objects in flow he would have found the mechanism proving his theory. He didn't. Nor would others until the mid-19th century. Fluid *instability* remained an undiscovered concept. In spite of the obvious burblings of water around stream rocks, that most flows, even fictionally defined frictionless incompressible flows, won't stay laminar or smoothly follow surfaces remained the opposite of 17th-and-18th-century formal analysis assumptions. This meant Newton couldn't know the mechanisms by which his theory might be shown as correct. Nor could the mathematicians who proved him wrong.

Oddly, their 'proofs,' based on mistaken assumptions about real fluids, introduced the single most useful equation for engineering purposes in the history of fluid dynamics and aerodynamics. That history is amusingly or sadly convoluted, with too many opportunities lost.

Newton's theory of inertially caused non-viscous pressure drag

Isaac Newton, in the three editions of his *Mathematical Principles of Natural Philosophy,* or, *The Principia* (1687, 1713, 1726) had asserted his theory of inertially caused pressure resistance (drag) between object and flow. He asserted that this drag would make fore-aft pressure differences even in "infinitely fluid mediums" (inviscid, or frictionless flows), as well as making a non-viscous component of drag within real fluids – fluids with the viscous shear frictions he also described.

Key to his theory was his assertion that a projectile will be slowed by fore-aft pressure differences, which also "arouse motion in the fluid." This is Newton's momentum exchange between object and flow, present in any drag. ("Motion" and "quantity of motion" were Newton's term for 'momentum.' Terms with double quotes are Newton's.) Newton built his theory from Descartes' and Huygens' conservation of momentum, also expressed by his three laws of motion, and from his experiments with objects sinking in water and falling through air.

And while it's possible to say what 'must be' merely by conservation of momentum, momentum is not a force. By Newton's first and second laws, the change of momentum (slowing of the projectile, acceleration of the wake) requires an external force. Newton defined the two external forces that operate on fluids, viscous shear "friction" and "pressure." For the force instrument of his non-viscous inertial resistance Newton correctly invoked "pressures."

Yet critically for his theory's non-survival, although eddies were observable behind any bridge pylon or stream rock, Newton never investigated the wake disturbances behind objects in flow or their drag effects. Newton focused on the fact of momentum exchange between moving object and wake rather than resultant eddy formation.

That Newton didn't investigate such swirls is a mystery, and was pivotal for fluid dynamics. Such wake disturbances are among multiple mechanisms of Newton's non-viscous inertially caused pressure drag.

In the simplest of modern analyses, formation of turbulence, separated flows, and vortices absorbs momentum and kinetic energy, which inertially persist until well behind an object in flow, and is thus unavailable for conversion to pressure recovery immediately behind the object. The difference between ambient or raised pressures ahead and lowered pressures behind makes Newton's non-viscous inertially caused pressure drag.

However, Newton's theory was sufficiently general that when in the mid-19th century its specific drag mechanisms (flow instabilities) finally started to get discovered, they fit right in. Or would have. But by then it was too late. His theory was long superseded, ignored, or dismissed. Nobody looked for its truths and thus didn't find them.

A few ways turbulence, flow separations, and wake vortices, make Newtonian non-viscous inertial drag

There are a few means by which turbulence, flow separations, cavitations, and wake vortices make Newtonian non-viscous inertial object-flow pressure drag. They show that Newton was correct to invoke inertia:

• First, any momentum that goes into formation of wake disturbances and inertially persists is then unavailable to be converted into pressure recovery immediately behind an object in flow, making Newton's unequal pressures fore and aft.

In real low-viscosity flows like air and water, it takes a while for viscous shear frictions to damp the inertia of flow disturbances. The rotational angular momentum of turbulence and vortices inertially persists until damped by viscosity, well downstream in low-viscosity fluids. All but the slowest (lowest Reynolds number, *Re)* real flows over objects are unstable and will develop turbulence and perhaps other instabilities.

In a fictional inviscid fluid there is no shear friction to damp formation of instabilities over objects; turbulence *always* forms, and perhaps flow separations and wake vortices. Without viscous damping, linear and rotational momentums of separated flows and disturbances inertially persist. More linear, separated flows may slow and increase in pressure as they collide with slower flows further back, but also may form vortices.

Note that an unstable flow is like balancing a pencil on its point. The pencil will always fall over. Similarly, if a flow is unstable it will always develop turbulence, and perhaps flow separations and wake vortices.

• Second, when flows separate from an object they generally have higher than ambient flow velocities. The curve of flows around objects centrifuges lower pressures to their sides. The pressure gradient from ambient or raised pressures ahead to centrifugally lowered pressures to the sides of the object increases local flow velocities. Separated flows carry this raised kinetic energy into wake rather than into pressure recovery.

Note that flows separate from a surface when their inertia exceeds the centripetal force of ambient pressures.

• Third, cavitations and partial vacuum pre-cavitations (history later.) For our purposes, in a cavitation, ambient or lowered pressures behind a moving object are insufficient to accelerate *inertial* fluids rapidly enough to catch up with the object, or to neck in fluids behind the object. In a gas, a strong vacuum forms. In a liquid, there are multiple stages of cavitation: vapors leaving solution to form bubbles, phase change of the liquid to gas, and strong vacuum formation.

The drop in pressure behind an object at lower speeds could be called a pre-cavitation partial vacuum: pressure energy is used up accelerating inertial fluid into the space behind the object, leaving a lowered pressure there.

• Fourth, separated flows add velocity to wake flows immediately behind the object. In 1955, Alexander Lippisch showed that the boundary between such separated flows and the 'dead' wake region behind the object is turbulent, mixing momentum into the wake behind, and dragging it backward.[5] That further lowers pressures behind the object.

• Fifth, at moderate speeds, separated flows will curl around the low pressure behind the object, creating von Kármán alternating vortices. The momentum of the separated flows is conserved as aftward momentum of the vortices, which alternately rip free making pulses of low pressure. See Figures 1 and 2.

• Sixth, separated flows don't converge to raise pressures behind an object. Behind streamlined shapes at subsonic speeds, flows remain attached and converge for pressure recovery aft, sometimes nearly to pressures ahead of the object, for low drag.

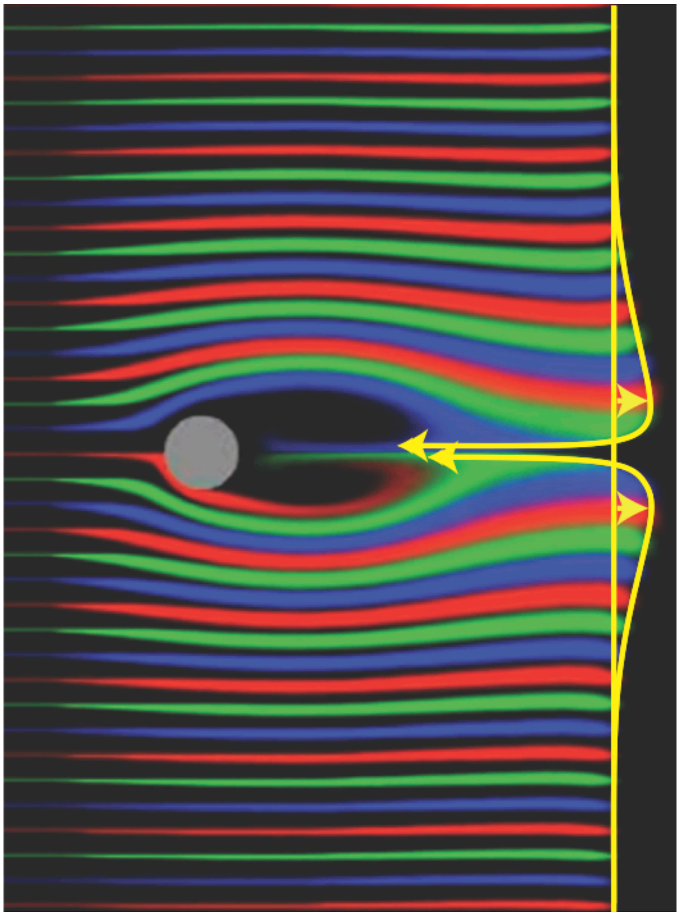


Figure 1: Initial flows. In this simulation, flow has just started around a cylinder, showing a displacement pattern Frederick William Lanchester diagrammed in 1907: slowed or reversed core wake flows balanced by fast outer-wake flows.[6] The vertical line shows flow if there were no object. Here side-to-side instability is just starting to form, with one vortex larger than the other. At moderate speeds, as flows evolve from the simplest pattern, side-to-side symmetry is unstable and will give way to alternating Kármán vortices.[7] Note that to the right where flows thicken they have slowed; we are looking at a raised-pressure wave, which adds a little to the axial forward pressure gradient.

• Seventh, in real fluids, viscous shear frictions can raise or lower non-viscous inertially caused pressure drag. Though not the only cause, shear frictions within thin 'boundary layers' over surfaces may trip flow separation and resulting vortices. But the cause of flow perturbations is still flow instability rather than viscous shear frictions, which tend to damp and delay turbulence. While friction and instabilities have an effect on each other, they are separate forms of drag.

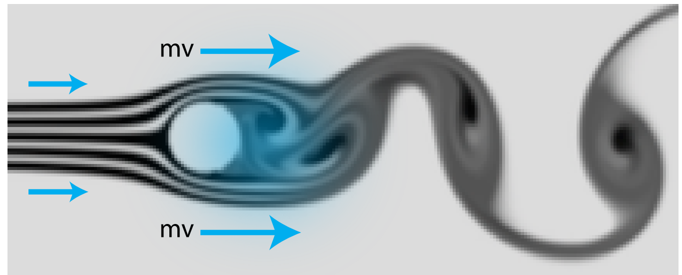


Figure 2:[8] Flow separation drag. Flow velocities increase to the sides of the object. Separated flows inertially maintain their velocity, momentum (mv), and kinetic energy as they become wake. The wake's inertia forms a partial vacuum behind the object (blue shading), around which flows may alternately curl, forming vortices centripetally held together around their centrifuged low-pressure centers. The aftward momentum of the separated flows doesn't disappear as it winds into a vortex, which thus has net downstream momentum that rips it free. Each ripping free reforms a partial vacuum behind the object, around which the opposite high-speed separated flow curves, forming a new vortex of opposite rotation. As the vortices peel off they form a Kármán alternating vortex 'street.' Most of the aft low pressures that 'pull' back on the object and pull forward on wake are within a few diameters of the object. Note that this wake volume is inertially 'sealed' against intrusions of higher-pressure fluid. Swirls further back are kinetic energy dumped into wake.

All of the above can also be stated in terms of kinetic energy. Kinetic energy ensconced in turbulence, separated flows, or vortices is unavailable to raise pressures aft. But energy analysis was not available in the 17th or 18th centuries, even though by 1686 Gottfried Wilhelm von Leibniz had formulated the precursor to energy, the mysteriously conserved *vis viva,* 'living force,' mv^2. Newton contentiously stuck with the conservation of momentum. Thomas Young used the term, 'energy,' in 1802, but it wouldn't enter common use until later in the century.

Gustave Gaspard de Coriolis established the work form of energy, force x distance, in 1828 -1829. Coriolis derived the modern $mv^2/2$ from redefining *vis viva* as weight x height.[9] William Thomson, Lord Kelvin, in 1849 - 1851 coined the term, 'kinetic energy.' In 1852 Michael Rankine integrated the various forms of energy, asserting they were convertible and conserved.[10]

Opportunities lost.

Had Newton investigated the eddies behind objects in flow, he unavoidably would have realized that formation of eddies soaks up momentum that 'must' subtract momentum from the object, for drag: he made a very similar analysis explaining the increased drag from a sinking object's oscillations, asserting that momentum that goes into the oscillation of a sinking object must come from the linear momentum of the object.[11] He then would have shifted from his general theory to its specific mechanisms. Fluid dynamics would have leapt forward by centuries.

The course of fluid dynamics has been as unstable as its flows. In the vicissitudes of history, Leonardo da Vinci's scientific works were unavailable for nearly three centuries after his 1519 death. Had they remained of scientific influence, his extensive studies of water and sketches of turbulent eddies behind objects in streams might have influenced Newton to investigate.

In the history of science, what is not formally studied might as well not exist. The unstable nature of flows, flow disturbance patterns, and resultant drag wouldn't start to be formally studied until 1842 and Sir George Gabriel Stokes. 'Instabilities' refers to both the unstable nature of flows and to the disturbance patterns that result.

Flow instability drag only belatedly studied.

In 1842, Sir George Gabriel Stokes showed that steady flow solutions to the Navier-Stokes equations (1821 – 1845) are not the only solutions and that eddies might develop.[12]

Studies of wave, atmospheric, and flow instabilities and resultant drag by Stokes, Poncelet, Saint-Venant, Kirchhoff, Helmholtz, Rayleigh, Taylor, Kelvin, and others followed, as progress with dead ends. Stokes initiated and abandoned an idea of 'deadwater wakes,' which nevertheless persisted into the WWI years in British attempts to mathematically model wing lift and drag.[13] In 1883 Osborne Reynolds derived a ratio of inertial to viscous forces useful in the prediction of transition from laminar (smooth) to higher-drag turbulent flows, now known as Reynolds numbers, *Re.* Understanding of the unstable nature of flows, both real and idealized, was more complete. The study of the mechanisms of drag from flow separations and instabilities has continued into this century. Instability drag was not applied to Newton's inertial drag theory.

The defeat of Newton's theory by d'Alembert's paradox

Within two decades of Newton's 1727 death, Jean le Rond d'Alembert (French) mathematically proved zero drag around a fore-aft symmetrical bluff object under false assumptions that real fluids were inviscid and that flows around such objects are smoothly fore-aft symmetrical in pattern, velocity, and pressure. Equal pressures fore and aft meant zero drag. A proof by Leonhard Euler (Swiss) was flawed but used the same assumptions. Each added the simplifying condition of incompressibility.

D'Alembert's analysis

D'Alembert's analysis was based on his independent derivation of what Euler would formalize in 1752 as the Bernoulli equation, which asserts a lossless exchange of fluid pressure, velocity, and fluid elevation *along* streamlines, implying zero exchange between object and flow:

$p + \frac{1}{2} \rho v^2 + \rho g h$ = constant along streamlines

Traditional Bernoulli is in terms easily measurable for engineering, pressure (p), density (rho, ρ), velocity (v), gravity (g), and height (h).

D'Alembert searched for the causes of drag in three papers, of 1744, 1752, and 1768. Since there is resistance, in 1768 d'Alembert declared his paradox.[14]

And then the unexamined axioms that founded fluid dynamics were carried into the future.

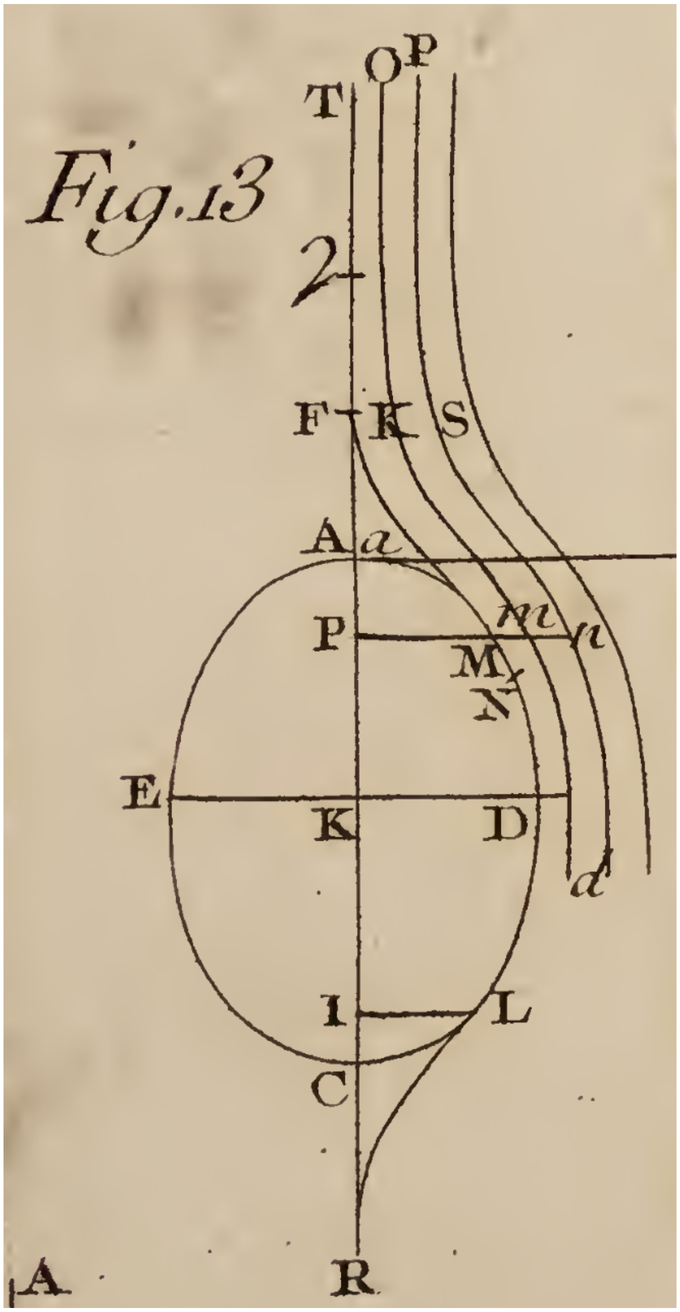


Figure 3: D'Alembert's paradox. His 1752 sketch implies fore-aft symmetrical patterns of flow, velocity, and by his independent derivation of the Bernoulli equation, pressure, for zero drag.[15] *He falsely assumed real fluids are frictionless. And he didn't have the 19th-century concept of unstable flow. The modern simplifying constraint, 'steady flow,' hides inertially caused pressure forces that otherwise would make this flow pattern asymmetrical and turbulent.*

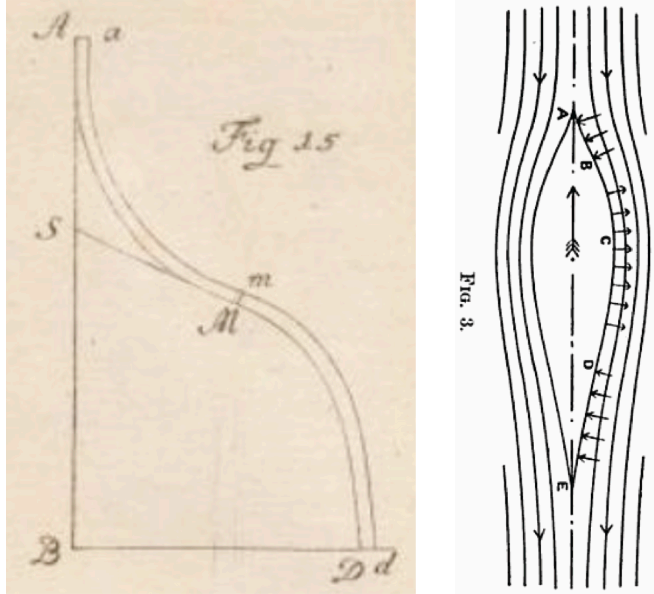


Figure 4: Centrifuging of pressures: In Euler's 1745 diagram (left) he incorrectly asserted zero velocity change from ahead to the side of the object, and thus no pressure change for no drag, apparently assuming fore-aft symmetry. But he did hint at centrifuging of pressures, asserting that pressures would only be raised between 'a' and 'm.' He didn't consider that this would make a pressure gradient along streamlines, nor did he consider the centrifugal lowering of pressures from 'm' to 'd,' strengthening that gradient and accelerating flows to the object's sides. Frederick William Lanchester's 1907 diagram (right) shows centrifuging of raised pressures ahead and aft, and centrifugally lowered pressures and narrowed streamtubes (higher velocities) to the sides.[16]

Euler's 'proof' of zero inviscid drag

Euler was becoming the dominant mathematician of his century, so his paradox 'proof' would have carried weight. But his proof, in his 1745 *Commentary* on a superb 1742 work by Benjamin Robins,[17] can mostly be ignored.

Euler's 'proof' does contain a prequel to perhaps the most neglected equation in applied aerodynamics, the equation for the centrifuging of pressures, which is a simplified but more explicit form of his equation for forces normal to streamlines, from his 1752 equations of inviscid fluid dynamics.[18]

And Euler did diagram ‘canals’ of flow around an object, now called streamtubes, perhaps after a 1736 figure by Daniel Bernoulli.[19] Streamlines don’t have volume, so they can’t carry momentum. Streamtubes are the volume-momentum-energy form of streamlines, with infinitesimal cross-sectional area.

But Euler’s ‘proof?’Euler diagrammed his ‘canal’ streamtubes around only the fore half of an object and claimed their flows wouldn’t change velocity from well ahead to the sides of the object (very false except in very slow flows), implying no change in pressure, and thus no drag.[20]

Twice ‘proven’ wrong

Having been twice ‘proven’ wrong, Newton’s theory of inertial pressure resistance was ignored, when even considered.

Future geometers (mathematicians) would claim d’Alembert’s paradox ‘resolved’ by each newly recognized form of drag. The viscous shear frictions of the Navier-Stokes equations of 1821 – 1845 were overemphasized. Later theorists studied flow separation patterns, turbulence, instabilities, without recognizing them as the mechanisms of Newton’s inertial pressure drag. It’s still common to hear the resolution of d’Alembert’s paradox attributed to shear frictions, and only rarely to unstable flows.

The evolution of simplifying conditions from early false assumptions about real fluids

Modern theorists usually write that d’Alembert and Euler derived their proofs and Bernoulli equations within ‘perfect fluid’ conditions: incompressible, frictionless, ‘steady’ (and ‘irrotational,’ beyond this paper) flows with no flow separations. But those are modern conditions recognized as simplifications to keep computations simple.

In modern terms, the ‘steady flow’ constraint requires, for computational simplicity, that flows not be allowed to evolve from d’Alembert’s and Euler’s laminar attached flows into turbulence, flow separations from surfaces, or vortices.

D’Alembert and Euler didn’t specify steady flows, as the opposite concept, unstable flows, didn’t exist. Their diagrams show an assumption of steady, laminar, attached flows that are fore-aft symmetrical around their fore-aft symmetrical objects. Such flow conditions are unstable and will devolve into the instabilities that provide Newton’s fore-aft pressure differences, for drag,[21] unless fictionally prevented by the modern simplifying condition, ‘steady flows.’

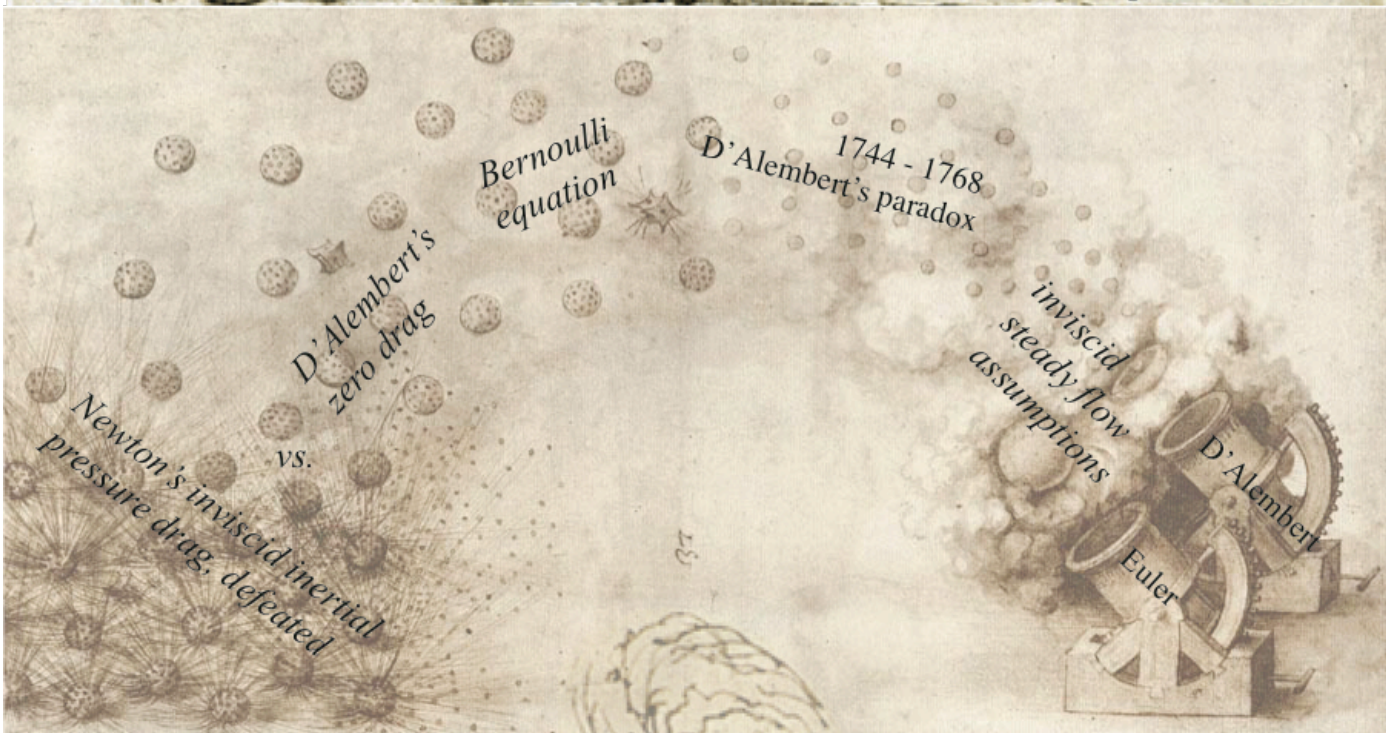
Both mathematicians knew their models didn’t match reality. Each recognized the paradox of actual fluid drag versus, as Euler put it, “their very great resistance.”[22]

D’Alembert and Euler either hadn’t read Newton’s descriptions of viscous shear frictions, disbelieved them, considered them insignificant, or only trusted their own analyses. Newton’s study of fluid “frictions” (viscous shear frictions) and “tenacity” (viscosity) would mostly be ignored until re-derived in the Navier-Stokes equations of 1821 – 1845.

For d’Alembert and especially Euler, the inviscid condition was an assumption about real fluids rather than a simplifying model. This is illustrated by Euler’s criticism of the English artillerist Benjamin Robins’ 1742 assertion that ball spin on an axis crosswise to travel caused veer (lift), which would require fluid viscosity. Robins had read Newton’s similar description of the curve of tennis balls.[23] Robins’ experimental results were decisive yet defeated by Euler, who asserted irregularities in manufacturing.[24] A century later Gustav Magnus, in a convincing experiment with flawed explanations, verified Robins’ conclusions in what should be called ‘the Robins effect’ rather than ‘the Magnus effect.’

Figure 5 (following page): Timeline of drag theories. Background art by Leonardo da Vinci, 1452 -1519.[25][26][27]

Leonardo da Vinci's 1508 - 1513 flow sketches are scattered until the 19th century. If not lost, would they have influenced Newton to study flow separation and wake eddies the missing mechanisms of:
Newton's 1687 theory of inviscid resistance (R) from inertial *(I)* pressures (P)?



1821 - 1845 Navier-Stokes equations.
Drag becomes overly attributed to viscosity.

1842 Stokes derives unstable flows

1842 - 21st-century:
D'Alembert's assumptions that real fluids are inviscid and 'steady' are recognized as useful fictions.

Realistic instability drag slowly develops, but isn't recognized as the mechanism of Newton's inertially-caused pressure drag, until:

2020 Newton's theory of inertial pressure resistance, here with help from da Vinci's sketches, *rises from the dead.*

In his *Commentary* on Robins' 1842 *New Principles of Gunnery,* Euler had attempted to resolve the paradox by a more formal treatment of: (1) Robins' experimental observation that at and above transonic speeds, air resistance on musket balls nearly triples; (2) Robins' conclusions that the jump in drag happens as a vacuum forms behind the ball (cavitation); and (3) that cavitation depends on projectile velocity and the pressure of the fluid.[28] Euler was unable to extend this cavitation analysis to partial vacuum drag behind subsonic objects.[29] The earlier inertial argument would have sufficed.

Since the fictional 'steady flow' constraint precludes the wake perturbations that are the mechanisms of Newton's non-viscous drag, under fictional 'perfect fluid' idealizations Newton's approach yields d'Alembert's zero drag. That's a case of fictions in, useful fictions out. Newton's theory is the more general, and yields results required by any parameters. For real conditions its predictions are real.

Caveats: D'Alembert did consider frictions, but didn't pursue the analysis as a resolution of his paradox. Darrigol observes that in 1749 he evoked velocity-proportional fluid-surface friction, and fluid *ténacité,* viscosity. And in 1744, noting the mathematical zero drag, "d'Alembert evoked the observed stagnancy of the fluid behind the body to retain only the Bernoulli pressure on the prow."[30] Thus he came close to investigating the evolution of unstable flow patterns. And then he retreated to his mathematical analyses.

The ignorance of the times

In the 18th century, there was no theoretical way for d'Alembert or Euler to formally know their smoothly attached depictions of fore-aft symmetrical flows were unstable and could only make zero drag if fictionally not allowed to evolve to flow separations, eddies, turbulence, and asymmetry.

Nor could Newton have known that a specific explanation rather than a general prediction of his inertial drag required the formally uninvestigated concept of unstable flows.

Under only apparently similar inviscid flow conditions, Newton's inertial pressure theory predicted drag while d'Alembert predicted zero drag. A mathematical proof of zero drag vs. a verbally described assertion of drag? Newton's theory lost.

Modern dismissiveness of Newton's theory

When (rarely) discussed, Newton's inertial resistance is dismissed:

> . . . there is no such thing as a component of resistance resulting from the inertia of the fluid! As d'Alembert showed . . . the resistance . . . when moving in an incompressible, inviscid fluid is *exactly zero.*[31] --George E. Smith, "Was Wrong Newton Bad Newton?" 2005.

Note that Smith left out d'Alembert's and Euler's pivotal false assumption, of 'steady,' smoothly attached flows.

Even in the definitive 1999 translation of the third edition of *Principia* from its original Latin, the authors of the 370-page preface only lament, "The loss of interest in Newton's theoretical model of inertial resistance . . ." because it meant the loss of interest in his timing of spheres dropped 220' from the newly constructed dome of St. Paul's Cathedral in London, "the first accurate measures of resistance forces."[32]

Attempts to apply instability drag to d'Alembert's paradox but not to Newton's theory.

> Another way to solve d'Alembert's paradox was to assume some instability of the laminar flow of a slightly-viscous fluid that prompted turbulent eddying in the rear of the body. Stokes first suggested this option in 1843. Poncelet and Saint-Venant made it the basis of quantitative resistance estimates... [Starting in 1846.]
>
> Their theories nonetheless lacked predictive power… –Olivier Darrigol, *Worlds of Flow.*[33]

Figure 6 (next page): Historical stages of drag theory. Kármán Vortex simulation image permission from Tintschl BioEnergie.[34] Da Vinci sketches.[35][36] Newton portrait.[37]

Stages of drag theory:

1: Early: Flows impart force. Moving objects slow within fluid.
All species experience drag.
384 – 322 BC: Aristotle on 'resistance.'

1519: da Vinci's fluid studiess, lost for centuries.

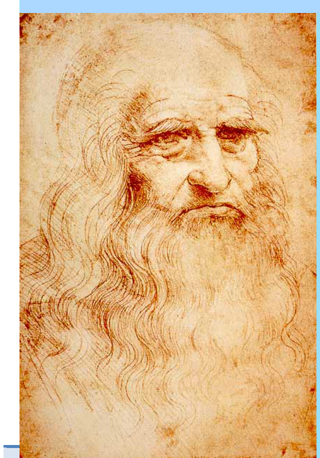
Drag effects on fluid are not formally considered, until:

2: 1687: Newton: Friction and pressure drag on object puts an opposite force on wake. No wake details.

Prescient but ignored: via observation, experiment, and his 3 laws of motion, Newton asserts that wake inertia makes a lower pressure behind an object in flow than ahead, without investigating the turbulence, wake instabilities, and flow separations that are its mechanisms.

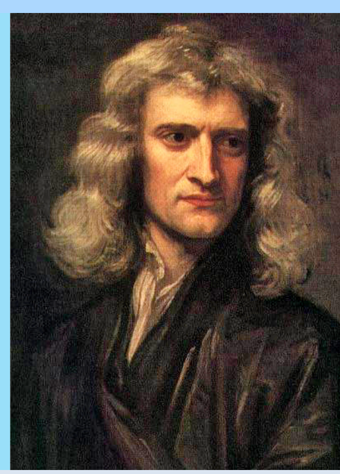
Progress and obstruction:
1744 - 1768: D'Alembert's paradox and Euler's Bernoulli equation derive zero drag, defeating Newton's theory of inertial pressure resistance. Their assumptions about real flows are incorrect.
1821 - 1845: NS equations re-derive Newton's sheer frictions, 'solving' the paradox (partially), but with over-emphasis on frictions.

3: 1842, Stokes:
Unstable flows!
Eddies!



5: 2020: Recognition of wake disturbances as the mechanisms of Newton's theory of inertially-caused pressure resistance allows the theory's reinstatement.

Leonardo da Vinci and Isaac Newton, the two greats of early fluid theory, whose contributions were largely lost for centuries or considered disproved and ignored.



Historically, conceptual descriptions that are difficult to quantitatively model seldom carried weight. That's a limitation of mathematical modeling rather than of conceptual descriptions, which might have salvaged Newton's theory.

Newton's motion exchange and d'Alembert's paradox. Anomaly and traps.

D'Alembert's paradox can be restated in terms of Newton's exchange: If there is no exchange of momentum between object and flow there is no drag. Ditto for kinetic energy.

Newton's exchange is verified by his second and third laws:

$F_{Drag} = m\Delta v_{Object}/sec = - m\Delta v_{Fluid}/sec = -F_{Fluid}$

This says, 'The pressure and friction drag forces on the object equals its change in momentum/time which is equal and opposite to the fluid's change of momentum/time which is caused by the pressure gradient and friction forces on the fluid. Forces on the object and fluid are equal and opposite.'

The middle terms are the 'ma' from Newton's F = ma, with ma as the change (Δ) in momentum (mΔv) per second.

This repeats Newton's third law of "motion" (momentum), that for every force (here drag on an object) there is an equal and opposite force (on the fluid).

A Newtonian drag exchange term for Bernoulli

The Bernoulli equation is recognized as simplistic. Several complications are usually explicitly ignored, mainly for engineering simplicity, or as having minimal impact on most calculations. These include losses to entropy (including flow disturbances), enthalpy (heat loss), and work, a form of energy. This 'work' is usually ignored, but shouldn't be: it is precisely a Newtonian exchange of energy between object and flow, for Newtonian inertially caused pressure drag.

Adding a Newton object-flow drag exchange term to the Bernoulli equation allows non-constant energy along streamtubes, immersed bow and stern waves with local energy concentrations, includes the possibility of Newtonian inertial drag, and has wind tunnel implications.

$(pAd + ½mv^2 + mgh)_{\text{flow field}} – (½mv^2 + mgh)_{object} –$ etc. = constant

This says that drag can suck kinetic energy out of the object, which slows, and adds kinetic energy to wake flows. (Or vice versa.) Energy is thus not constant along streamtubes if there is inviscid drag, or any drag.

To get more complete we could add a thrust energy term. Then for an object maintained at constant velocity, energy added by thrust would end up in the flow field as wake disturbance.

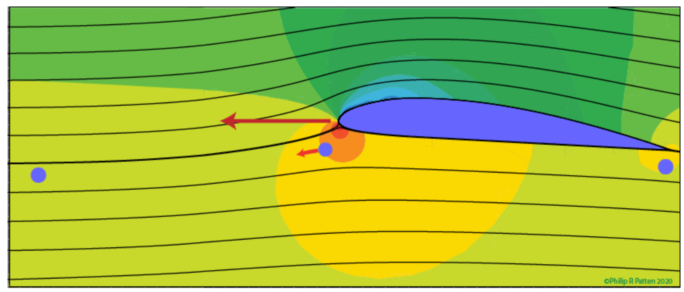


Figure 7: Local concentration of energy and the Bernoulli anomaly. A wing moving to the left approaches a still parcel of air (left blue circle). As a wing passes over the previously undisturbed parcel, the raised pressure below its forward stagnation point pushes it forward while its pressure is increased. Its kinetic and pressure energies increase simultaneously, a local total energy concentration, for non-constant energy along streamlines, opposite of what the simplistic Bernoulli equation predicts.

The Bernoulli anomaly perspective trap

The simple Bernoulli equation shows constant energy along streamlines (more accurately, along streamtubes) around an object in flow. That's true under perfect fluid conditions, but in real flows, Newton's drag momentum/energy exchange requires local concentrations of total energy along streamtubes. Object kinetic energy lost to drag adds to local concentrations of kinetic plus pressure energies along streamtubes.

	Newton	D'Alembert	Euler	Conclusions:
Basis	3 laws of motion, experiment, observation	Bernoulli & incomplete mathematical model	Bernoulli and partly false conceptual model	
17th-18th-century correct conditions	Viscous vs. **inviscid fluids.** Flow **asymmetry** implied by drag 'motion' exchange	X	X	**N's hypothetical inviscid fluid with unknown unstable flow condition…**
17th-18th-century false assumptions.		Real flows assumed **steady, inviscid,** and **symmetrical**	Real flows assumed **steady, inviscid,** and **symmetrical**	**seems similar to D & E's false inviscid steady symmetrical flow assumptions.**
Simplifying conditions	Incompressible fluids	Incompressible fluids	Incompressible fluids	
Unknown in the 17th-18th-centuries	Flow instability as the hidden drag mechanism	Symmetrical flows are unstable	Symmetrical flows are unstable	
17th-18th-century results	**Inertially caused non-viscous pressure drag plus friction drag.** Object-flow momentum exchange.	**Zero drag.**	**Zero drag.**	**Different results from apparently similar inviscid steady conditions**
19th-21st-century conditions	Unstable flows except in the fictional 'steady' flow simplification	Fictional perfect fluid simplification. Instabilities disallowed.	Fictional perfect fluid simplification. Instabilities disallowed.	
19th-21st-century similar results in fictional perfect fluid conditions, (instabilities not allowed).	**Zero drag (when instabilities not allowed).** Constant energy along streamlines.	**Zero drag.** Constant energy along streamlines.	**Zero drag.** Constant energy along streamlines.	**Similar results under fictional 'perfect fluid' 'steady flow' conditions**
19th-21st-century results in non-perfect fluid and real conditions.!	**Inertially caused instability pressure drag.** Local energy concentrations.	X	X	**Newton shows different results when instabilities are allowed.**

Figure 8 (previous page): Newton's, d'Alembert's, and Euler's period conditions and conclusions versus modern conditions and conclusions.

The usual wind tunnel perspective hides an old and generally ignored Bernoulli anomaly around wings and hides the fact of local energy concentrations along streamlines: Below the forward stagnation point of a wing in lift, where upper and lower flows separate, there is a raised-pressure area. As a tiny 'parcel' of flow that will pass under the wing approaches this area from ahead along a streamline, it slows. That's a Bernoulli exchange of increased pressure for decreased velocity.

But that's deceptive. From the perspective of a moving wing disturbing previously still air, as that raised-pressure region approaches the previously still parcel of air the wing will pass over, the parcel is within increasing pressures that push it forward, though not as fast as the wing. *Pressure and velocity are increased simultaneously,* for raised local total energy and an 'immersed bow pressure wave.'

Similarly, total energy is raised along streamtubes in the raised-pressure volume just ahead of a moving sphere.

Terminal velocities show Newtonian momentum/energy drag exchange

Where there is drag, even in inviscid but otherwise real fluids, thrust is required to maintain velocity. Any limited thrust on an object results in limited displacement pressure gradients around the object that can only accelerate an *inertial* fluid out of the way of the object to limited velocities. At terminal velocities, drag equals thrust.

At terminal velocities, thrust energy (force x distance) doesn't go into increasing object kinetic energy, but via drag energy lost (again force x distance) goes into accelerating wake, validating Newton's 'motion' exchange in 'non-perfect' flows.

This doesn't hold within 'perfect fluid' models. Within fictional inviscid, steady flow, incompressible conditions, ambient pressure acts as if infinite, cavitation can never happen, flows around a fore-aft symmetrical body remain fore-aft symmetrical, pressure gradients fore and aft can symmetrically raise toward infinity and still be balanced for zero drag, and object velocity can increase toward infinity. Simple Bernoulli is perfectly accurate. Under these idealized conditions, d'Alembert's zero-drag holds. But for engineering purposes, the simple Bernoulli equation's predictions of flows and pressures are often close enough to be calibrated.

Hiding Newton's exchange: the wind tunnel perspective trap

The wind-tunnel perspective trap: In a steady-state diagram, or relative to an observer at a wind tunnel, an object or model has zero momentum. It's still. So how can there be a Newtonian momentum exchange? And the flow is in a mean steady state, even with turbulence and fluctuations, so how can it be exchanging momentum and energy with a stationary model?

Where an object is held at constant velocity by thrust, there are two Newtonian momentum exchanges: thrust adds momentum to the object, and drag subtracts equal momentum from the object and adds it to wake.

This holds for energy exchanges also: In a wind tunnel, the force from the mount on the model times the distance it travels relative to airflow is work, energy. That's by F x d = work. The opposite force, drag times distance relative to airflow, is also work. Energy thus added to the object is subtracted from it by drag and added to wake.

So there is a pair of Newtonian momentum or energy exchanges operating in steady states. It's addition and equal subtraction.

Newton's motion exchange is more obvious as in his example, where an object is free to slow.

Summary

The course of early fluid dynamics was as unstable as its flows. Three turning points were: 1: The loss of da Vinci's studies of turbulent flows, affecting; 2: that Newton didn't investigate the flow separations, wake eddies, and their effect on drag that could have proved his theory of inertially caused pressure resistance, a non-viscous, separable component of drag in real flows and operant in inviscid flows, and; 3: his theory's defeat by d'Alembert's and

Euler's 'proof' of zero-drag. Newton showed his theory at a general level but didn't find the turbulence, flow separation, and eddy formation that are its specific mechanisms. Each absorbs and inertially holds momentum, which then can't convert into raised pressures behind an object, making the fore-aft pressure imbalance that Newton predicted. Such flow instabilities weren't formally investigated until after 1842, so d'Alembert's 1744 proof of zero drag under inviscid conditions seemed similar to Newton's inviscid conditions and appeared to disprove his theory. 'Steady and inviscid flow conditions' started as d'Alembert's and Euler's incorrect assumptions about real fluids and evolved into modern fictional conditions simplifying engineering approximations. 'Steady' flow is the modern fictional modeling constraint under which flow separations and perturbations are not allowed. Along with incompressibility, inviscid and 'steady' define the 'perfect fluid' idealization and the only conditions under which d'Alembert's and the Bernoulli equation's predictions of zero drag hold. Within d'Alembert's equivalent of 'perfect' flow conditions, the mechanisms of Newton's inertial pressure drag can't exist, so Newton's theory also predicts D'Alembert's zero drag. All but the slowest (lowest Reynolds number) real flows are unstable, and around surfaces will develop turbulence and perhaps flow separations and instabilities, for a Newtonian non-viscous component of drag. Without the damping effect of viscosity, and without the 'steady' flow constraint, all theoretical inviscid flows over objects are unstable and will always develop turbulence and perhaps other flow disturbances, for Newtonian inviscid drag. Newton's theory of non-viscous inertially caused pressure drag is correct under all conditions and is reinstated.

Newton's ghost says, "Q.E.D."

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